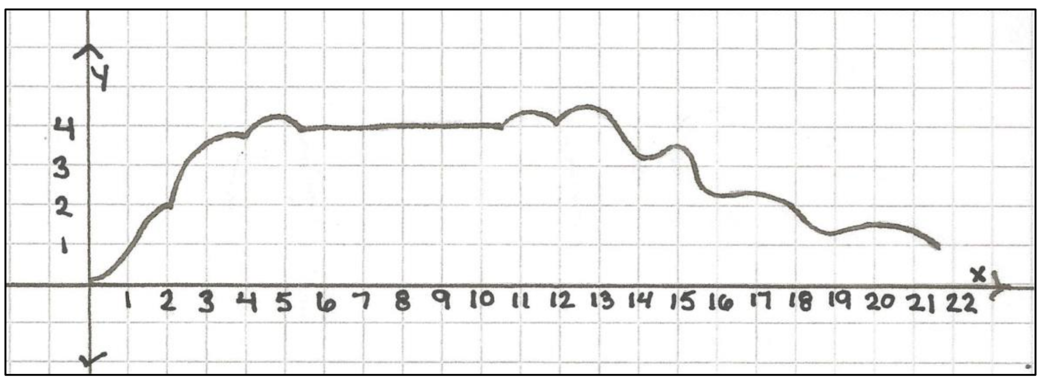
SHINING CHANDELIER

HISTORY:

When walking into the Dominican Center and looking up, there is a beautiful chandelier that is very important to Mount St. Mary College history. In appreciation to Bishop John Dunn, the Republic of Armenia gave the chandelier to the college as a gift. Bishop Dunn was deeply involved in helping others. One of his charitable works included helping the Dominican Sisters buy the property which the college was built on. Another one of his works included helping the Armenian refugees who arrived in New York City after WWI. Many of these families came to New York with only the clothes on their backs and their children. To help these families Bishop Dunn found them jobs, apartments, and their children schools. Though he worked in the city and had his funeral at St. Patrick's Cathedral, Bishop Dunn asked to be buried in Newburgh, the town that he loved. His grave is in the Sisters' cemetery next to the Dominican Center.

PROBLEM:

An outline of a part of the chandelier is given. Use solids of revolution to find the volume obtained by rotating this curve around the x-axis. After finding the volume of the solid, assume the graph is 5.4375 inches long and the chandelier is 46 inches long; find the volume of the chandelier.

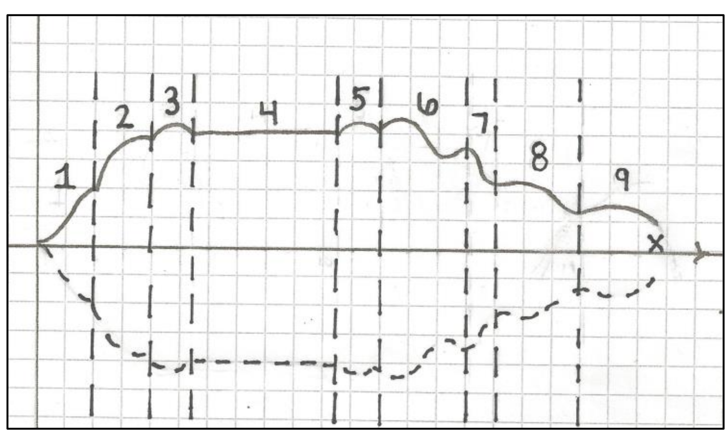


Not drawn to scale

Then approximate the volume of the chandelier by using the volume of a cone, assuming the width of the chandelier is 33inches, and compare both volumes calculated.

The following functions can be used to model the curve:

1. y = -cos(x) + 1
2. y = -$x^2 + 7x - 8.25$
3. y = -2$x^2 + 18x - 36.5$
4. y = 4
5. y = -2$x^2 + 43.28x - 230.1148$
6. y = -2cos$(.6x - 14) + 5$
7. y = -.5cos$(-3x + 5) + 2.7$
8. y = -.9cos$(.9x) + 2.5$
9. y = -.5$(x - 20.6)^2 + 1.3$



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An outline of the chandelier is given. Use solids of revolution to find the volume obtained by rotating this curve around the x-axis. After finding the volume of the solid, assume the graph is 5.4375 inches long and the chandelier is 46 inches long; find the volume of the chandelier. Then approximate the volume by using the volume of a cone, assuming width of the chandelier is 33inches, and compare both volumes calculated.

SOLUTION:

First we can use each equation as the radius of a circle in a circle area problem and integrate to find how much volume there would be when each function is rotated around the x-axis.

1. $\int_0^2 \pi(-\cos(x)+1)^2 = 0.992\pi$
2. $\int_2^4 \pi\,(-x^2+7x-8.25)^2 = 24.192\pi$
3. $\int_4^{5.5} \pi(-2x^2+18x-36.5)^2 = 18.825\pi$
4. $\int_{5.5}^{10.5} \pi(4)^2 = 80\pi$
5. $\int_{10.5}^{12} \pi(-2x^2+43.28x-230.1448)^2 = 16.895\pi$
6. $\int_{12}^{15} \pi(-2\cos(.6x-14)+5)^2 = 34.904\pi$
7. $\int_{15}^{16} \pi(-.5\cos(-3x+5)+2.7)^2 = 8.836\pi$
8. $\int_{16}^{19} \pi(-.9\cos(.9x)+2.5)^2 = 29.905\pi$
9. $\int_{19}^{22} \pi(-.5(x-20.6)^2+1.3)^2 = 2.899\pi$

Now we can add the sections together to get the volume of the solid region created by the revolution of the graph.

$0.992\pi + 24.192\pi + 18.825\pi + 80\pi + 16.895\pi + 34.904\pi + 8.836\pi + 29.905\pi + 2.899\pi$

Volume of the solid $\approx 217.448\pi$

Now we set up a ratio to find what the "actual" volume is.

$$\frac{5.4375in}{217.448\pi\ in^3} = \frac{46\ in}{V}$$

5.4375* V = 10,002.608 π

V = 1,839.56 $\pi \approx$ 5,779.15 in^3

The "actual" volume of the chandelier is approximately 5,779.15in^3.

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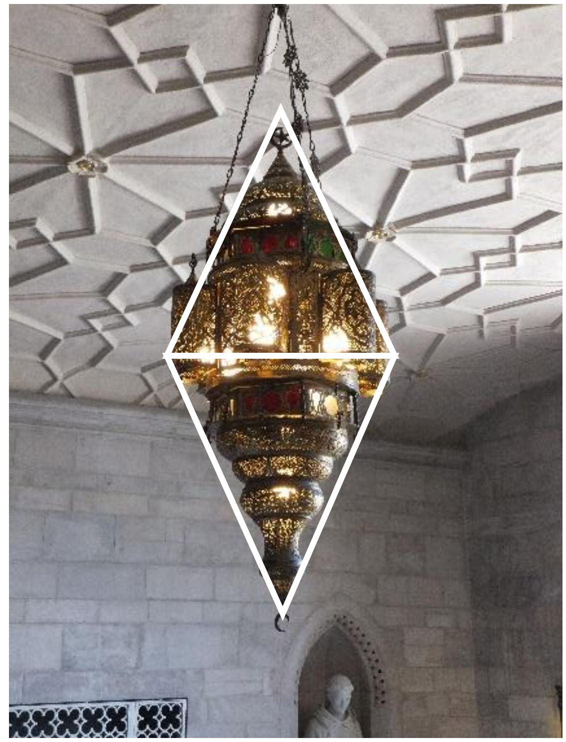
Then approximate the volume by using the volume of a cone, assuming width of the chandelier is 33inches, and compare both volumes calculated.



The volume can be approximated with two cones of the same size with their bases together.

Volume of two cones = $2(\pi r^2 \frac{h}{3})$

Take half the full height for the for the height of the cones and half the width for the radius

h = 46/2 = 23 inches

r = 33/2 = 16.5 inches

V = $2(\pi 16.5^2 \frac{23}{3})$

V = 2 (2087.25π)

V = 13114.58 in^3

13,114.58 in^3 – 5,779.15 in^3 = 7,335.43 in^3

There is a difference of 7,335.43in^3 .